

## A higher-derivative Lee-Wick standard model

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## A higher-derivative Lee-Wick standard model

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**ABSTRACT:** The Lee-Wick Standard Model assumes a minimal set of higher-derivative quadratic terms that produce a negative-norm partner for each Standard Model particle. Here we introduce additional terms of one higher order in the derivative expansion that give each Standard Model particle two Lee-Wick partners: one with negative and one with positive norm. These states collectively cancel unwanted quadratic divergences and resolve the hierarchy problem as in the minimal theory. We show how this next-to-minimal higher-derivative theory may be reformulated via an auxiliary field approach and written as a Lagrangian with interactions of dimension four or less. This mapping provides a convenient framework for studies of the formal and phenomenological properties of the theory.

**KEYWORDS:** Beyond Standard Model, Higgs Physics.

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## 1. Introduction

Extensions of the Standard Model (SM) generally involve mass scales that are much higher than the scale of electroweak symmetry breaking. If one views the SM as a low-energy effective theory, then the Higgs boson squared mass  $m_h^2$  receives radiative corrections that grow quadratically with the cutoff. This leads to the hierarchy problem: A large separation of scales requires an extremely close cancellation between the bare Higgs boson mass and the cutoff-dependent loop corrections. Within the low-energy effective theory, such a fine tuning has no natural explanation.

Solutions to the hierarchy problem can be grouped into three broad categories, distinguished by their assumptions: (1) models that assume fine tuning to be extreme and present, but natural from the point of view of the string landscape, as in split-supersymmetric models [1]; (2) models that assume fine tuning is not extreme since no high mass scales are present, as in scenarios with large extra dimensions and a low Planck scale [2]; (3) models that assume fine tuning is not extreme, even when high mass scales are present, because new physics just above the electroweak scale modifies the ultraviolet divergence of  $m_h^2$  from quadratic to logarithmic. The Minimal Supersymmetric Standard Model (MSSM) is perhaps the most famous example of a model in the last category: Each SM particle has a supersymmetric partner with the same gauge quantum numbers but opposite spin

statistics. As fermion and boson loops enter with opposite relative signs, quadratic divergences cancel between Feynman loop diagrams when both particles and their associated superpartners are taken into account.

A similar cancellation is achieved in the Lee-Wick Standard Model (LWSM) [3], which has recently been proposed as a theory that solves the hierarchy problem. Each SM particle possesses a Lee-Wick (LW) partner [4] with the *same* spin statistics, but with opposite-sign quadratic terms. Since the propagators of ordinary and LW particles differ in overall sign, quadratic divergences cancel between pairs of diagrams. A LW partner for a given field arises via the inclusion of a higher-derivative (HD) kinetic term which generates an additional pole in the associated two-point function. As reviewed below, the HD Lagrangian can be recast, using auxiliary fields, as a dimension-four Lagrangian that includes partner fields with “wrong-sign” quadratic terms [3]. The cancellation of divergences in this formulation of the theory occurs because HD terms in the original Lagrangian cause propagators to fall off more quickly with momentum, so that loop diagrams become less divergent.

While LW particles have wrong-sign kinetic and mass terms (like Pauli-Villars regulators) it is nonetheless believed consistent to treat them as physical particles. Neither the LWSM [3], in which all the LW states can decay, nor the  $O(N)$  LW model at large  $N$  [5] violates causality at a macroscopic level. Moreover, studies of longitudinal gauge-boson scattering in the LWSM indicate that unitarity is not violated provided the HD theory can be mapped to a Lagrangian with interactions of dimension four or less [6]. Taking these observations into account, a number of authors have begun to explore the phenomenology [7, 8] and cosmology [9] of LW extensions of the SM. These studies have assumed the minimal theory, in which the lowest-order HD term for each field is included, and precisely one LW partner accompanies each SM particle.

While the minimal scenario is the simplest to study, one may wonder whether the inclusion of a single HD term, and *exactly* no others of higher order, represents a natural state of affairs. In this paper we explore a next-to-minimal scenario that includes HD terms of the next order in a derivative expansion, leading to *two* partners for each SM particle. Our immediate focus is a technical one: What is the generalization of the auxiliary field (AF) formulation introduced in the minimal theory [3], and what form of the HD Lagrangian leads to an auxiliary field theory with interactions of dimension four or less? We address this question in a non-Abelian gauge theory with fermions and complex scalars, so that our results can be immediately applied to the SM. Interestingly, one of the two new LW partners for each SM particle is ordinary (with correct-sign quadratic terms), suggesting that collider signatures and experimental limits on this theory can be qualitatively different from the minimal version. Our results suggest that there is no impediment, in principle, to constructing similar theories with additional LW states via the inclusion of appropriate interactions that are of yet higher order in the number of derivatives.

We note that previous work [10, 11] extensively studies a particular  $O(p^6)$  form for a HD scalar Lagrangian, in which  $O(p^4)$  terms are absent and gauge couplings are omitted. In particular, this work develops a strongly-interacting Higgs sector that tames ultraviolet corrections and can be studied on the lattice. Reference [10] represents pioneering early work on the consistency of  $O(p^6)$  scalar theories. By contrast, the thrust here is to study

the duality between more general HD theories with  $O(p^6)$  terms and equivalent theories with operators of dimension four or less, not only in the Higgs sector but including all SM particles, with an eye toward future phenomenological studies.

This paper is organized as follows. In the next section we review the LW idea in a simple scalar field theory and show how the AF formulation is applied when HD terms of next-to-lowest order are present. In section 3 we extend our approach to non-Abelian gauge theories, focusing on the pure gauge sector; in section 4 we show how fermions are included in the theory. In section 5 we discuss the Higgs sector of the theory. In section 6 we discuss the cancellation of one-loop quadratic divergences in an  $SU(N_c)$  gauge theory with complex scalars and chiral fermions. In section 7 we summarize our conclusions.

## 2. A scalar example

Let us begin by reviewing the formulation of a LW theory of a real scalar field. The simplest HD Lagrangian is given by

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2}\hat{\phi}\square\hat{\phi} - \frac{1}{2M^2}\hat{\phi}\square^2\hat{\phi} - \frac{1}{2}m_\phi^2\hat{\phi}^2 + \mathcal{L}_{\text{int}}(\hat{\phi}), \quad (2.1)$$

where the last term represents interactions. The HD term leads to an additional pole in the  $\hat{\phi}$  two-point function near the mass  $M$ , which corresponds to the LW partner of the usual state with mass eigenvalue near  $m_\phi$ . The HD term also assures high-momentum falloff of the  $\hat{\phi}$  propagator as  $1/p^4$ , improving the convergence of  $\hat{\phi}$  loop diagrams. Following the approach of ref. [3], one observes that eq. (2.1) is equivalent to a Lagrangian including an auxiliary field,  $\tilde{\phi}$  and no higher-derivative interactions:

$$\mathcal{L}_{\text{AF}} = -\frac{1}{2}\hat{\phi}\square\hat{\phi} - \frac{1}{2}m_\phi^2\hat{\phi}^2 - \tilde{\phi}\square\hat{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 + \mathcal{L}_{\text{int}}(\hat{\phi}). \quad (2.2)$$

The  $\tilde{\phi}$  equation of motion (EOM) is

$$\tilde{\phi} = \frac{1}{M^2}\square\hat{\phi}, \quad (2.3)$$

which, upon substitution into eq. (2.2), reproduces the original Lagrangian of eq. (2.1). The kinetic terms in eq. (2.2) can be diagonalized via the substitution

$$\hat{\phi} = \phi - \tilde{\phi}, \quad (2.4)$$

yielding

$$\mathcal{L} = -\frac{1}{2}\phi\square\phi + \frac{1}{2}\tilde{\phi}\square\tilde{\phi} - \frac{1}{2}m_\phi^2(\phi - \tilde{\phi})^2 + \frac{1}{2}M^2\tilde{\phi}^2 + \mathcal{L}_{\text{int}}(\phi - \tilde{\phi}). \quad (2.5)$$

The scalar mass matrix can be diagonalized without affecting the form of the kinetic terms via a symplectic transformation:

$$\begin{pmatrix} \phi \\ \tilde{\phi} \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} \phi_0 \\ \tilde{\phi}_0 \end{pmatrix}, \quad (2.6)$$

where the subscript 0 indicates a mass eigenstate; one finds

$$\tanh 2\theta = \frac{-2m_\phi^2}{M^2 - 2m_\phi^2}. \quad (2.7)$$

The final Lagrangian takes the form

$$\mathcal{L}_{\text{LW}} = -\frac{1}{2}\phi_0 \square \phi_0 + \frac{1}{2}\tilde{\phi}_0 \square \tilde{\phi}_0 - \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{2}M_0^2\tilde{\phi}_0^2 + \mathcal{L}_{\text{int}}[e^{-\theta}(\phi_0 - \tilde{\phi}_0)], \quad (2.8)$$

where  $m_0$  and  $M_0$  are the mass eigenvalues, and the factor of  $e^{-\theta}$  can be absorbed into redefinitions of the couplings. The opposite-sign  $\phi_0$  and  $\tilde{\phi}_0$  propagators following from the quadratic terms in eq. (2.8), together with the specific relationship between the  $\phi_0$  and  $\tilde{\phi}_0$  couplings in  $\mathcal{L}_{\text{int}}$ , assures the cancellation of quadratic divergences, as is shown explicitly in ref. [3].

Indicating by  $N$  the number of physical poles in the  $\hat{\phi}$  propagator, let us refer to the minimal example just considered as an  $N = 2$  theory. An  $N = 3$  model corresponds to a HD Lagrangian of the general form

$$\mathcal{L}_{\text{HD}}^{N=3} = -\frac{1}{2}\hat{\phi} \square \hat{\phi} - \frac{1}{2M_1^2}\hat{\phi} \square^2 \hat{\phi} - \frac{1}{2M_2^4}\hat{\phi} \square^3 \hat{\phi} - \frac{1}{2}m_\phi^2\hat{\phi}^2 + \mathcal{L}_{\text{int}}(\hat{\phi}), \quad (2.9)$$

where  $M_1$  and  $M_2$  are the LW mass scales, which we assume are comparable. The restriction that the  $\hat{\phi}$  propagator has three physical poles restricts the values of  $m_\phi^2$ ,  $M_1^2$  and  $M_2^2$ , so that it is possible to map eq. (2.9) to a Lagrangian of the form

$$\mathcal{L}_{\text{LW}}^{N=3} = \sum_{i=1}^3 c_i \left[ -\frac{1}{2}\phi^{(i)}(\square + m_i^2)\phi^{(i)} \right] + \mathcal{L}_{\text{int}}(\{\phi^{(i)}\}), \quad (2.10)$$

where the  $c_i = 1$  or  $-1$ , and the  $m_i^2$  are positive. The missing link that connects eq. (2.9) to (2.10) is an AF Lagrangian, analogous to eq. (2.2) in the  $N = 2$  theory, and appropriate field redefinitions, analogous to eq. (2.4). Let us first examine the special case where  $m_\phi = 0$  [which corresponds to  $m_1 = 0$  in eq. (2.10)] before stating the general result. The desired AF Lagrangian involves two new scalar fields,  $\chi$  and  $\psi$ :

$$\mathcal{L}_{\text{AF}} = -\frac{1}{2}\hat{\phi} \square \hat{\phi} - \chi \square \hat{\phi} + m_2 m_3 \chi \psi - \frac{1}{2}\psi \square \psi - \frac{1}{2}(m_2^2 + m_3^2)\psi^2 + \mathcal{L}_{\text{int}}(\hat{\phi}). \quad (2.11)$$

Like the field  $\tilde{\phi}$  in the  $N = 2$  theory,  $\chi$  is an auxiliary field; since it occurs linearly in eq. (2.11), its EOM imposes a constraint that is exact at the quantum level:

$$\psi = \frac{1}{m_2 m_3} \square \hat{\phi}. \quad (2.12)$$

Substituting eq. (2.12) into eq. (2.11), one obtains

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2}\hat{\phi} \square \hat{\phi} - \frac{1}{2} \left( \frac{m_2^2 + m_3^2}{m_2^2 m_3^2} \right) \hat{\phi} \square^2 \hat{\phi} - \frac{1}{2} \left( \frac{1}{m_2^2 m_3^2} \right) \hat{\phi} \square^3 \hat{\phi} + \mathcal{L}_{\text{int}}(\hat{\phi}), \quad (2.13)$$

which factorizes as

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2m_2^2 m_3^2} \hat{\phi} \square (\square + m_2^2) (\square + m_3^2) \hat{\phi} + \mathcal{L}_{\text{int}}(\hat{\phi}), \quad (2.14)$$

and from which one identifies  $m_\phi = 0$ ,  $M_1^2 = m_2^2 m_3^2 / (m_2^2 + m_3^2)$  and  $M_2^4 = m_2^2 m_3^2$  upon comparison with eq. (2.9).

Showing next that the AF Lagrangian can also be written in the form of eq. (2.10) is a simple matter of linear algebra. Taking  $m_2$  to be the lighter LW state and substituting the field redefinitions

$$\hat{\phi} = \phi^{(1)} - \frac{m_3}{(m_3^2 - m_2^2)^{1/2}} \phi^{(2)} + \frac{m_2}{(m_3^2 - m_2^2)^{1/2}} \phi^{(3)}, \quad (2.15)$$

$$\chi = \frac{1}{(m_3^2 - m_2^2)^{1/2}} [m_3 \phi^{(2)} - m_2 \phi^{(3)}], \quad (2.16)$$

$$\psi = \frac{1}{(m_3^2 - m_2^2)^{1/2}} [m_2 \phi^{(2)} - m_3 \phi^{(3)}], \quad (2.17)$$

into eq. (2.11), one obtains

$$\mathcal{L} = -\frac{1}{2} \phi^{(1)} \square \phi^{(1)} + \frac{1}{2} \phi^{(2)} (\square + m_2^2) \phi^{(2)} - \frac{1}{2} \phi^{(3)} (\square + m_3^2) \phi^{(3)} + \mathcal{L}_{\text{int}}(\hat{\phi}). \quad (2.18)$$

As with eq. (2.4) in the  $N=2$  theory, eq. (2.15) leads to a very specific form for the interaction terms in eq. (2.18). We find that there is no finite field redefinition that takes the AF Lagrangian eq. (2.11) to the LW form eq. (2.18) for  $m_2 = m_3$ , so we do not consider that possibility further.

For completeness, we exhibit the results for  $m_\phi$  (and  $m_1$ ) non-zero. The AF Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{\text{AF}} = \frac{1}{\eta_1} \left[ -\frac{1}{2} \hat{\phi} (\square + m_1^2) \hat{\phi} - \chi (\square + m_1^2) \hat{\phi} + (m_3^2 - m_1^2)^{1/2} (m_2^2 - m_1^2)^{1/2} \chi \psi \right. \\ \left. - \frac{1}{2} \psi \square \psi - \frac{1}{2} (m_2^2 + m_3^2 - m_1^2) \psi^2 \right] + \mathcal{L}_{\text{int}}(\hat{\phi}), \end{aligned} \quad (2.19)$$

where  $\eta_1 \equiv (m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2) / (m_2^2 - m_1^2) (m_3^2 - m_1^2)$ . Varying eq. (2.19) with respect to auxiliary field  $\chi$  generalizes the EOM eq. (2.12) to

$$\psi = \frac{1}{(m_2^2 - m_1^2)^{1/2} (m_3^2 - m_1^2)^{1/2}} (\square + m_1^2) \hat{\phi}, \quad (2.20)$$

which, when substituted back into eq. (2.19), yields

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2\Lambda^4} \hat{\phi} (\square + m_1^2) (\square + m_2^2) (\square + m_3^2) \hat{\phi}, \quad (2.21)$$

where

$$\Lambda^4 \equiv m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2. \quad (2.22)$$

Equation (2.21) is equivalent to the HD Lagrangian in eq. (2.9) with the identifications

$$m_\phi^2 = (m_1^2 m_2^2 m_3^2) / \Lambda^4, \tag{2.23}$$

$$M_1^2 = \Lambda^4 / (m_1^2 + m_2^2 + m_3^2), \tag{2.24}$$

$$M_2^2 = \Lambda^2. \tag{2.25}$$

On the other hand, one can obtain the canonical LW form, eq. (2.10) with  $c_1 = -c_2 = c_3 = 1$ , from eq. (2.19) by the field redefinitions

$$\hat{\phi} = \sqrt{\eta_1} \phi^{(1)} - \sqrt{-\eta_2} \phi^{(2)} + \sqrt{\eta_3} \phi^{(3)}, \tag{2.26}$$

$$\chi = \sqrt{-\eta_2} \phi^{(2)} - \sqrt{\eta_3} \phi^{(3)}, \tag{2.27}$$

$$\psi = \sqrt{\eta_3} \phi^{(2)} - \sqrt{-\eta_2} \phi^{(3)}, \tag{2.28}$$

where the parameters  $\eta_i$  are defined by

$$\eta_1 \equiv \frac{\Lambda^4}{(m_2^2 - m_1^2)(m_3^2 - m_1^2)}, \tag{2.29}$$

$$\eta_2 \equiv \frac{\Lambda^4}{(m_1^2 - m_2^2)(m_3^2 - m_2^2)}, \tag{2.30}$$

$$\eta_3 \equiv \frac{\Lambda^4}{(m_1^2 - m_3^2)(m_2^2 - m_3^2)}. \tag{2.31}$$

Noting, for example, that  $\eta_1 = 1$  when  $m_1 = 0$ , one sees that eqs. (2.15)–(2.17) immediately follow in this case. As before, we assume  $m_3 > m_2 > m_1$ , so that  $\text{sign}(\eta_i) = (-1)^{i+1}$ . The remarkable algebraic simplifications that occur in converting the AF Lagrangian are a consequence of simple sum rules that are satisfied by the  $\eta_i$ :

$$\sum_{i=1}^3 m_i^{2n} \eta_i = 0 \quad (n = 0, 1), \tag{2.32}$$

$$\sum_{i=1}^3 m_i^{2n} \eta_i = \Lambda^4 \quad (n = 2), \tag{2.33}$$

$$m_1^2 m_2^2 \eta_3 + m_2^2 m_3^2 \eta_1 + m_3^2 m_1^2 \eta_2 = \Lambda^4. \tag{2.34}$$

Our  $\eta_i$  parameters are equivalent to those introduced by Pais and Uhlenbeck [12] (which we call  $\eta_i^{\text{PU}}$ ) to describe purely quantum-mechanical theories with HD Lagrangians analogous to those used here. The mapping

$$\eta_i = \frac{m_i^4 \Lambda^{2N-2}}{\prod_j m_j^2} \eta_i^{\text{PU}} \tag{2.35}$$

converts the sum rules of ref. [12] into eqs. (2.32) and (2.34) for the case  $N = 3$ , while eq. (2.33) is linearly dependent on the others.

The interaction terms in the general  $N = 3$  theory are functions of  $\hat{\phi}$ . Following from eq. (2.26),

$$\mathcal{L}_{\text{int}}(\hat{\phi}) \equiv \mathcal{L}_{\text{int}} \left( \sqrt{\eta_1} \phi^{(1)} - \sqrt{-\eta_2} \phi^{(2)} + \sqrt{\eta_3} \phi^{(3)} \right). \tag{2.36}$$



The restriction on the form of the couplings imposed by eq. (2.36) is necessary for the cancellation of divergences. This fact is illustrated in the following simple example: Let  $\mathcal{L}_{\text{int}}(\hat{\phi}) = \lambda \hat{\phi}^4/4!$ , or equivalently,

$$\mathcal{L}_{\text{int}}(\hat{\phi}) = \frac{\lambda}{4!} \sum_{ijkl} \sqrt{|\eta_i \eta_j \eta_k \eta_l|} \phi^{(i)} \phi^{(j)} \phi^{(k)} \phi^{(l)} . \quad (2.37)$$

The self-energy for  $\phi^{(1)}$  (corresponding to the state that is present when the LW particles are decoupled) is given by

$$\Pi(p^2) = \lambda \eta_1 \int \frac{d^4 p}{(2\pi)^4} \sum_k \left[ \frac{(-1)^{k+1} i}{p^2 - m_k^2} \right] |\eta_k| , \quad (2.38)$$

where the factor  $(-1)^{k+1}$  yields the appropriate overall sign for each scalar propagator. Using the fact that  $(-1)^{k+1} |\eta_k| = \eta_k$  and formally expanding the integrand, one finds

$$\Pi(p^2) = i \lambda \eta_1 \int \frac{d^4 p}{(2\pi)^4} \sum_k \left( \frac{\eta_k}{p^2} + \frac{\eta_k m_k^2}{p^4} + \frac{\eta_k m_k^4}{p^6} + \dots \right) . \quad (2.39)$$

The first two terms vanish as a consequence of the  $n = 0$  and  $1$  sum rules, eq. (2.32), respectively; these terms would otherwise be quadratically and logarithmically divergent, respectively. Although the interactions in the LW form of the  $N = 3$  theory are more complicated than in the  $N = 2$  case, the sum rules satisfied by the  $\eta_i$  always provide the necessary algebraic miracles that cancel the leading divergences in the theory.<sup>1</sup>

### 3. Pure Yang-Mills theory

We now generalize the approach of the previous section to a pure Yang-Mills theory. The next-to-leading-order HD Lagrangian reads

$$\mathcal{L}_{\text{HD}} = -\frac{1}{2} \text{Tr} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \left( \frac{1}{m_2^2} + \frac{1}{m_3^2} \right) \text{Tr} \hat{F}_{\mu\nu} \hat{D}^\mu \hat{D}_\alpha \hat{F}^{\alpha\nu} - \frac{1}{m_2^2 m_3^2} \text{Tr} \hat{F}_{\mu\nu} \hat{D}^\mu \hat{D}_\alpha \hat{D}^{[\alpha} \hat{D}_\beta \hat{F}^{\beta\nu]} , \quad (3.1)$$

where the superscript brackets indicate antisymmetrization of just the first and last indices:

$$X^{[\alpha_1 \alpha_2 \dots \alpha_{N-1} \alpha_N]} \equiv X^{\alpha_1 \alpha_2 \dots \alpha_{N-1} \alpha_N} - X^{\alpha_N \alpha_2 \dots \alpha_{N-1} \alpha_1} . \quad (3.2)$$

Equation (3.1) can be written in the elegant factorized form

$$\mathcal{L}_{\text{HD}} = \text{Tr} \hat{F}_{\mu\nu} \left( \frac{1}{2} g^\mu{}_\alpha + \frac{\hat{D}^\mu \hat{D}_\alpha}{m_2^2} \right) \left[ \left( \frac{1}{2} g^\nu{}_\beta + \frac{\hat{D}^\nu \hat{D}_\beta}{m_3^2} \right) g^\alpha{}_\lambda - (\alpha \leftrightarrow \nu) \right] \hat{F}^{\beta\lambda} . \quad (3.3)$$

The field strength  $\hat{F}$ , and the covariant derivative  $\hat{D}$  acting upon a field  $X$  transforming in the adjoint representation of the gauge group, are defined in the usual manner:

$$\hat{F}^{\mu\nu} \equiv \partial^\mu \hat{A}^\nu - \partial^\nu \hat{A}^\mu - ig [\hat{A}^\mu, \hat{A}^\nu] , \quad (3.4)$$

$$\hat{D}^\mu X \equiv \partial^\mu X - ig [\hat{A}^\mu, X] . \quad (3.5)$$

---

<sup>1</sup>Despite this example,  $N > 2$  LWSMs are not finite theories, but remain logarithmically divergent, as can be shown by a generalization of the power-counting argument given in ref. [3].

This HD Lagrangian may be obtained from the equivalent Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{YM}} = & -\frac{1}{2} \text{Tr} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \text{Tr} \hat{F}^{\mu\nu} (\hat{D}_\mu \chi_\nu - \hat{D}_\nu \chi_\mu) - \frac{1}{2} \text{Tr} (\hat{D}_\mu \omega_\nu - \hat{D}_\nu \omega_\mu)^2 \\ & - 2m_2 m_3 \text{Tr} \chi_\mu \omega^\mu + (m_2^2 + m_3^2) \text{Tr} \omega_\mu \omega^\mu, \end{aligned} \quad (3.6)$$

where the new fields  $\chi$  and  $\omega$  transform in the adjoint representation. Integration by parts on the second term leads to a form for  $\mathcal{L}_{\text{YM}}$  in which no derivatives on  $\chi$  appear, making it an auxiliary field; since  $\chi$  appears linearly in  $\mathcal{L}_{\text{YM}}$ , it is also a Lagrange multiplier. The constraint imposed by its EOM,

$$\hat{D}_\nu \hat{F}^{\nu\mu} - m_2 m_3 \omega^\mu = 0, \quad (3.7)$$

is exact at the quantum level. Using eq. (3.7) to eliminate  $\omega^\mu$  from eq. (3.6), one finds that the terms proportional to  $\chi$  cancel, and that the remaining terms reduce to the HD Lagrangian, eq. (3.1).

In order to obtain a Lagrangian in the LW form, we rewrite the three fields  $\hat{A}$ ,  $\chi$  and  $\omega$  in terms of three new fields  $A_{1,2,3}$ :

$$\begin{aligned} A_1^\mu & \equiv \hat{A}^\mu + \chi^\mu, \\ A_2^\mu & \equiv \sqrt{-\frac{\eta_2}{\eta_1}} \chi^\mu - \sqrt{\frac{\eta_3}{\eta_1}} \omega^\mu, \\ A_3^\mu & \equiv \sqrt{\frac{\eta_3}{\eta_1}} \chi^\mu - \sqrt{-\frac{\eta_2}{\eta_1}} \omega^\mu. \end{aligned} \quad (3.8)$$

Under the action of the gauge group,  $A_2$  and  $A_3$  transform as matter fields in the adjoint representation, while  $A_1$  transforms as a gauge field, due to the additional shift in  $\hat{A}$ . The inverse transformations are given by

$$\begin{aligned} \hat{A}^\mu & = A_1^\mu - \sqrt{-\frac{\eta_2}{\eta_1}} A_2^\mu + \sqrt{\frac{\eta_3}{\eta_1}} A_3^\mu, \\ \chi^\mu & = \sqrt{-\frac{\eta_2}{\eta_1}} A_2^\mu - \sqrt{\frac{\eta_3}{\eta_1}} A_3^\mu, \\ \omega^\mu & = \sqrt{\frac{\eta_3}{\eta_1}} A_2^\mu - \sqrt{-\frac{\eta_2}{\eta_1}} A_3^\mu, \end{aligned} \quad (3.9)$$

as may be shown by using the sum rule eq. (2.32). Substituting eqs. (3.9) into eq. (3.6) is a laborious but straightforward procedure. Using eqs. (2.29)–(2.31) to express the parameters  $\eta_i$  in terms of masses  $m_{2,3}$ , and defining the unhatted field strength  $F_1^{\mu\nu}$  and covariant derivative  $D^\mu$  as analogous to eqs. (3.4)–(3.5) with  $\hat{A}^\mu \rightarrow A_1^\mu$ , one obtains the Lagrangian

$$\mathcal{L}_{\text{YM,LW}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2, \quad (3.10)$$

where the subscript indicates the power of  $g$  that appears in the coefficient of each gauge-invariant term. The kinetic and mass terms are contained in

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{2} \text{Tr} F_1^{\mu\nu} F_{1\mu\nu} + \frac{1}{2} \text{Tr} (D_\mu A_{2\nu} - D_\nu A_{2\mu})^2 - \frac{1}{2} \text{Tr} (D_\mu A_{3\nu} - D_\nu A_{3\mu})^2 \\ & - m_2^2 \text{Tr} A_2^\mu A_{2\mu} + m_3^2 \text{Tr} A_3^\mu A_{3\mu}, \end{aligned} \quad (3.11)$$

from which one immediately sees that  $A_1$  is massless ( $m_1=0$ ), and only  $A_2$  has wrong-sign quadratic terms,

$$\begin{aligned} \mathcal{L}_1 = & \frac{-ig}{m_3^2 - m_2^2} \text{Tr} (F_{1\mu\nu} [m_3 A_2^\mu - m_2 A_3^\mu, m_3 A_2^\nu - m_2 A_3^\nu]) \\ & + \frac{ig}{(m_3^2 - m_2^2)^{1/2}} \{ \text{Tr} (D_\mu A_{2\nu} - D_\nu A_{2\mu}) (2m_3 [A_2^\mu, A_2^\nu] - m_2 [A_2^\mu, A_3^\nu] - m_2 [A_3^\mu, A_2^\nu]) \\ & + \text{Tr} (D_\mu A_{3\nu} - D_\nu A_{3\mu}) (2m_2 [A_3^\mu, A_3^\nu] - m_3 [A_2^\mu, A_3^\nu] - m_3 [A_3^\mu, A_2^\nu]) \} , \end{aligned} \quad (3.12)$$

and finally,

$$\begin{aligned} \mathcal{L}_2 = & \frac{g^2}{2(m_3^2 - m_2^2)^2} \\ & \times \left\{ m_3^2 (4m_2^2 - 3m_3^2) \text{Tr} [A_2^\mu, A_2^\nu]^2 + 2m_2^2 m_3^2 \text{Tr} [A_2^\mu, A_2^\nu] [A_{3\mu}, A_{3\nu}] + m_2^2 (4m_3^2 - 3m_2^2) \text{Tr} [A_3^\mu, A_3^\nu]^2 \right. \\ & + 2m_2 m_3 (m_3^2 - 2m_2^2) \text{Tr} [A_2^\mu, A_2^\nu] ([A_{2\mu}, A_{3\nu}] + [A_{3\mu}, A_{2\nu}]) \\ & + 2m_2 m_3 (m_2^2 - 2m_3^2) \text{Tr} [A_3^\mu, A_3^\nu] ([A_{2\mu}, A_{3\nu}] + [A_{3\mu}, A_{2\nu}]) \\ & \left. + (m_2^4 - m_2^2 m_3^2 + m_3^4) \text{Tr} ([A_2^\mu, A_3^\nu] + [A_3^\mu, A_2^\nu]) ([A_{2\mu}, A_{3\nu}] + [A_{3\mu}, A_{2\nu}]) \right\} . \end{aligned} \quad (3.13)$$

While these expressions appear rather involved, they are substantially simpler than they could be, owing to the sum rules eqs. (2.32)–(2.34). Note that the decay  $A_3 \rightarrow A_2 A_1$  follows from the first term in  $\mathcal{L}_1$  since  $m_3 > m_2$ . In a complete theory, including fermions and Higgs fields, decay channels open for  $A_2$  as well.

#### 4. Fermions

The next-to-leading-order HD Lagrangian for a chiral fermion field  $\hat{\phi}_L$  assumes the compact form

$$\mathcal{L}_{\text{HD},f} = \frac{1}{m_2^2 m_3^2} \bar{\phi}_L \left[ (i\hat{\mathcal{D}})^2 - m_2^2 \right] \left[ (i\hat{\mathcal{D}})^2 - m_3^2 \right] i\hat{\mathcal{D}}\hat{\phi}_L , \quad (4.1)$$

where  $\hat{\mathcal{D}}$  includes both the gauge bosons and their LW partners. This HD Lagrangian may be obtained from the equivalent Lagrangian

$$\begin{aligned} \mathcal{L}_f = & \bar{\phi}_L i\hat{\mathcal{D}}\hat{\phi}_L - \bar{\chi}_R i\hat{\mathcal{D}}\chi_R + \bar{\psi}_L i\hat{\mathcal{D}}\psi_L + (\bar{\phi}_L i\hat{\mathcal{D}}\chi_L + \text{h.c.}) + (\bar{\chi}_R i\hat{\mathcal{D}}\psi_R + \text{h.c.}) \\ & + \frac{m_2 m_3}{m_2 + m_3} [\bar{\chi}_R (\chi_L + \psi_L) + \text{h.c.}] - (m_2 + m_3) (\bar{\psi}_L \psi_R + \text{h.c.}) . \end{aligned} \quad (4.2)$$

The fields  $\chi_L$  and  $\psi_R$ , which like  $\hat{\phi}_L$  are Weyl spinors transforming in the fundamental representation of the gauge group, appear only linearly in eq. (4.2), and therefore may be considered auxiliary. Varying  $\mathcal{L}_f$  with respect to them yields the constraints

$$i\hat{\mathcal{D}}\hat{\phi}_L + \frac{m_2 m_3}{m_2 + m_3} \chi_R = 0 , \quad (4.3)$$

$$i\hat{\mathcal{D}}\chi_R - (m_2 + m_3) \psi_L = 0 , \quad (4.4)$$

which may be substituted directly into  $\mathcal{L}_f$  to eliminate all terms linear in  $\chi_L$  and  $\psi_R$ , and also to re-express the the remaining fields  $\chi_R$ ,  $\psi_L$  in terms of  $\hat{\phi}_L$ :

$$\chi_R = -\frac{m_2 + m_3}{m_2 m_3} i \hat{\mathcal{D}} \hat{\phi}_L, \quad (4.5)$$

$$\psi_L = \frac{i \hat{\mathcal{D}}}{m_2 + m_3} \chi_R = -\frac{1}{m_2 m_3} (i \hat{\mathcal{D}})^2 \hat{\phi}_L, \quad (4.6)$$

where the final equality is obtained by substituting eq. (4.3) into eq. (4.4). It is straightforward to check that these EOMs transform eq. (4.2) into the HD form eq. (4.1).

In order to obtain a Lagrangian in the LW form, we rewrite the three left-handed fields  $\hat{\phi}_L$ ,  $\chi_L$  and  $\psi_L$  in terms of three new fields  $\phi_L^{(1,2,3)}$ , and the two right-handed fields  $\chi_R$ ,  $\psi_R$  in terms of two new fields  $\phi_R^{(2,3)}$ :

$$\begin{aligned} \phi_L^{(1)} &\equiv \hat{\phi}_L + \chi_L, \\ \phi_L^{(2)} &\equiv \sqrt{-\frac{\eta_2}{\eta_1}} \chi_L - \sqrt{\frac{\eta_3}{\eta_1}} \psi_L, \\ \phi_L^{(3)} &\equiv \sqrt{\frac{\eta_3}{\eta_1}} \chi_L - \sqrt{-\frac{\eta_2}{\eta_1}} \psi_L, \end{aligned} \quad (4.7)$$

and

$$\begin{aligned} \phi_R^{(2)} &\equiv \sqrt{-\frac{\eta_2}{\eta_1}} \chi_R - \left[ \sqrt{-\frac{\eta_2}{\eta_1}} + \sqrt{\frac{\eta_3}{\eta_1}} \right] \psi_R, \\ \phi_R^{(3)} &\equiv \sqrt{\frac{\eta_3}{\eta_1}} \chi_R - \left[ \sqrt{-\frac{\eta_2}{\eta_1}} + \sqrt{\frac{\eta_3}{\eta_1}} \right] \psi_R. \end{aligned} \quad (4.8)$$

The inverse transformations, whose simplification uses the sum rule eq. (2.32), are

$$\begin{aligned} \hat{\phi}_L &= \phi_L^{(1)} - \sqrt{-\frac{\eta_2}{\eta_1}} \phi_L^{(2)} + \sqrt{\frac{\eta_3}{\eta_1}} \phi_L^{(3)}, \\ \chi_L &= \sqrt{-\frac{\eta_2}{\eta_1}} \phi_L^{(2)} - \sqrt{\frac{\eta_3}{\eta_1}} \phi_L^{(3)}, \\ \psi_L &= \sqrt{\frac{\eta_3}{\eta_1}} \phi_L^{(2)} - \sqrt{-\frac{\eta_2}{\eta_1}} \phi_L^{(3)}, \end{aligned} \quad (4.9)$$

and

$$\begin{aligned} \chi_R &= \left[ \sqrt{-\frac{\eta_2}{\eta_1}} + \sqrt{\frac{\eta_3}{\eta_1}} \right] \left[ \phi_R^{(2)} - \phi_R^{(3)} \right], \\ \psi_R &= \sqrt{\frac{\eta_3}{\eta_1}} \phi_R^{(2)} - \sqrt{-\frac{\eta_2}{\eta_1}} \phi_R^{(3)}. \end{aligned} \quad (4.10)$$

Substituting these transformations into eq. (4.2) and using the sum rules eqs. (2.32)–(2.34) leads to a remarkable set of simplifications. Once the parameters  $\eta_i$  are expressed in terms of masses  $m_2$ ,  $m_3$ , the LW fermion Lagrangian reads

$$\mathcal{L}_{f, \text{LW}} = \bar{\phi}_L^{(1)} i \hat{\mathcal{D}} \phi_L^{(1)} - \bar{\phi}^{(2)} (i \hat{\mathcal{D}} - m_2) \phi^{(2)} + \bar{\phi}^{(3)} (i \hat{\mathcal{D}} - m_3) \phi^{(3)}, \quad (4.11)$$

where of course  $\phi \equiv \phi_L + \phi_R$ . Note from the signs of the terms that  $\phi^{(2)}$  and  $\phi^{(1),(3)}$  are negative- and positive-norm states, respectively. The HD, AF and LW Lagrangians for a right-handed chiral fermion field  $\hat{\phi}_R$  can be obtained from those presented here by the exchange  $R \leftrightarrow L$  throughout. The results can then be applied immediately to any chiral gauge theory (in particular, to the SM) without significant modification.

## 5. The Higgs sector

The discussion of the theory of a real scalar field in section 2 can be generalized in a straightforward way to one of a complex scalar  $\hat{H}$  that transforms in the fundamental representation of a non-Abelian gauge group. Let us first consider the case in which the squared scalar mass is positive,  $m_H^2 > 0$ . The HD Lagrangian may be written

$$\mathcal{L}_{\text{HD}} = \hat{D}_\mu \hat{H}^\dagger \hat{D}^\mu \hat{H} - m_H^2 \hat{H}^\dagger \hat{H} - \frac{1}{M_1^2} \hat{H}^\dagger (\hat{D}_\mu \hat{D}^\mu)^2 \hat{H} - \frac{1}{M_2^4} \hat{H}^\dagger (\hat{D}_\mu \hat{D}^\mu)^3 \hat{H} + \mathcal{L}_{\text{int}}(\hat{H}) \quad , \quad (5.1)$$

where  $m_H^2$ ,  $M_1^2$  and  $M_2^2$  are given by eqs. (2.22)–(2.25) with the identification  $m_\phi^2 = m_H^2$ . The auxiliary field Lagrangian analogous to eq. (2.19) is

$$\begin{aligned} \mathcal{L}_{\text{AF}} = & \frac{1}{\eta_1} \left\{ \hat{D}_\mu \hat{H}^\dagger \hat{D}^\mu \hat{H} - m_1^2 \hat{H}^\dagger \hat{H} - \left[ \chi^\dagger (\hat{D}_\mu \hat{D}^\mu + m_1^2) \hat{H} + \text{h.c.} \right] \right. \\ & \left. + (m_2^2 - m_1^2)^{1/2} (m_3^2 - m_1^2)^{1/2} (\chi^\dagger \psi + \psi^\dagger \chi) + \hat{D}_\mu \psi^\dagger \hat{D}^\mu \psi - (m_2^2 + m_3^2 - m_1^2) \psi^\dagger \psi \right\} \\ & + \mathcal{L}_{\text{int}}(\hat{H}) \quad , \end{aligned} \quad (5.2)$$

where  $\psi$  and the auxiliary field  $\chi$  also transform in the fundamental representation. Again, one recovers the HD form of the Lagrangian by applying the constraint equation obtained from varying with respect to  $\chi$ . The standard LW form of the theory is obtained via field redefinitions identical to eqs. (2.26)–(2.28), with the relabelling  $\hat{\phi} \rightarrow \hat{H}$  and  $\phi^{(i)} \rightarrow H^{(i)}$ :

$$\begin{aligned} \mathcal{L} = & -H^{(1)\dagger} (\hat{D}_\mu \hat{D}^\mu + m_1^2) H^{(1)} + H^{(2)\dagger} (\hat{D}_\mu \hat{D}^\mu + m_2^2) H^{(2)} \\ & - H^{(3)\dagger} (\hat{D}_\mu \hat{D}^\mu + m_3^2) H^{(3)} + \mathcal{L}_{\text{int}}(\hat{H}) \quad , \end{aligned} \quad (5.3)$$

where

$$\mathcal{L}_{\text{int}}(\hat{H}) = \mathcal{L} \left( \sqrt{\eta_1} H^{(1)} - \sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)} \right) \quad . \quad (5.4)$$

In the SM, spontaneous symmetry breaking is ensured by  $m_H^2 < 0$ . In this case it is more convenient to absorb the  $m_H^2$  term into  $\mathcal{L}_{\text{int}}$ :

$$\mathcal{L}_{\text{HD}} = \mathcal{L}_{\text{HD}}(m_H^2 = 0) + \mathcal{L}'_{\text{int}}(\hat{H}) \quad , \quad (5.5)$$

$$-\mathcal{L}'_{\text{int}}(\hat{H}) \equiv \frac{\lambda}{4} \left( \hat{H}^\dagger \hat{H} - \frac{v^2}{2} \right)^2 \quad , \quad (5.6)$$

where  $v$  is the Higgs vacuum expectation value. The mass parameters  $m_2$  and  $m_3$  are now determined by

$$M_1^2 = \frac{m_2^2 m_3^2}{m_2^2 + m_3^2} \quad \text{and} \quad M_2^2 = m_2 m_3 \quad . \quad (5.7)$$

The  $m_H^2 = 0$  part of the Lagrangian is handled via the steps described in section 2. Using the  $m_1^2 = 0$  values of the  $\eta_i$  parameters (and noting that  $\eta_1 = 1$ ), one then finds that the canonical LW form of the Higgs-sector Lagrangian is given by

$$\mathcal{L} = \hat{D}_\mu H^{(1)\dagger} \hat{D}^\mu H^{(1)} - \hat{D}_\mu H^{(2)\dagger} \hat{D}^\mu H^{(2)} + \hat{D}_\mu H^{(3)\dagger} \hat{D}^\mu H^{(3)} + m_2^2 H^{(2)\dagger} H^{(2)} - m_3^2 H^{(3)\dagger} H^{(3)} + \mathcal{L}'_{\text{int}} \left( H^{(1)} - \sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)} \right), \quad (5.8)$$

where the last term may be expanded

$$\begin{aligned} -\mathcal{L}'_{\text{int}} &= \frac{\lambda}{4} \left( H^{(1)\dagger} H^{(1)} - \frac{v^2}{2} \right)^2 + \frac{\lambda}{2} \left( H^{(1)\dagger} H^{(1)} - \frac{v^2}{2} \right) \\ &\times \left[ H^{(1)\dagger} (\sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)}) + \text{h.c.} \right] + |\sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)}|^2 \\ &+ \frac{\lambda}{4} \left\{ \left[ H^{(1)\dagger} (\sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)}) + \text{h.c.} \right] + |\sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)}|^2 \right\}^2. \end{aligned} \quad (5.9)$$

In analogy to the minimal theory [3], one may work in unitary gauge, in which

$$H^{(1)} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + h_1) \end{pmatrix}, \quad H^{(2)} = \begin{pmatrix} h_2^+ \\ \frac{1}{\sqrt{2}}(h_2 + iP_2) \end{pmatrix}, \quad H^{(3)} = \begin{pmatrix} h_3^+ \\ \frac{1}{\sqrt{2}}(h_3 + iP_3) \end{pmatrix}, \quad (5.10)$$

where the fields  $h_i$ ,  $P_i$  and  $h_i^+$  represent the scalar, pseudoscalar and charged Higgs components, respectively. Note that the mass terms in eq. (5.9) are given by

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= \frac{1}{2} m_2^2 (2h_2^- h_2^+ + h_2^2 + P_2^2) - \frac{1}{2} m_3^2 (2h_3^- h_3^+ + h_3^2 + P_3^2) \\ &- \frac{1}{2} m^2 (h_1 - \sqrt{-\eta_2} h_2 + \sqrt{\eta_3} h_3)^2, \end{aligned} \quad (5.11)$$

with  $m^2 = \lambda v^2/2$ , indicating that the charged and pseudoscalar Higgs masses are given directly by the parameters  $m_2$  and  $m_3$ . The neutral Higgs mass matrix, however, is off-diagonal; the mass eigenstate basis is obtained via a transformation that preserves the form of the neutral Higgs kinetic terms, which are proportional to  $\text{diag}(1, -1, 1)$ , in the basis  $(h_1, h_2, h_3)$ . Such transformation matrices can be found numerically, as was demonstrated, for example, in ref. [7]. Using such a numerical diagonalization, and the results presented here, one can study the phenomenology of the Higgs sector like any other multi-Higgs doublet extension of the SM. Derivation of the mass matrices of the LW gauge bosons and fermions is straightforward using the field redefinitions determined in this and the last two sections.

## 6. Application: divergence cancellation

In this section we consider the cancellation of divergences in an  $N = 3$   $SU(N_c)$  gauge theory with a single complex scalar field in the fundamental representation. This discussion generalizes the one appearing in section III of ref. [3], and provides a number of explicit calculations using the LW form of the theory. We also check that one-loop quadratic divergences cancel when chiral fermions are present.

One can learn much about the divergences of the theory by considering the HD form of the Lagrangian in Landau gauge, where the  $N = 3$  gauge boson propagator scales as  $p^{-6}$  at high energies ( $p$  denotes a generic momentum). The complex scalar propagator also scales as  $p^{-6}$ , while the Faddeev-Popov ghost propagator scales as  $p^{-2}$ . The salient issue is whether the derivatives at the new interaction vertices in the HD theory compensate for the additional momentum suppression in the propagators. In the  $N = 3$  theory, a vertex with  $n$  vectors scales as  $p^{8-n}$ , a vertex with two scalars and  $n$  vectors as  $p^{6-n}$ , and one with two ghosts and one gauge field as  $p$ . The steps for constructing the superficial degree of divergence,  $d$  are identical to those discussed in section III of ref. [3], so we do not repeat them. The result in the  $N = 2$  theory,

$$d = 6 - 2L - E - E' - 2E_g \quad (N = 2), \quad (6.1)$$

becomes

$$d = 8 - 4L - E - E' - 3E_g \quad (N = 3), \quad (6.2)$$

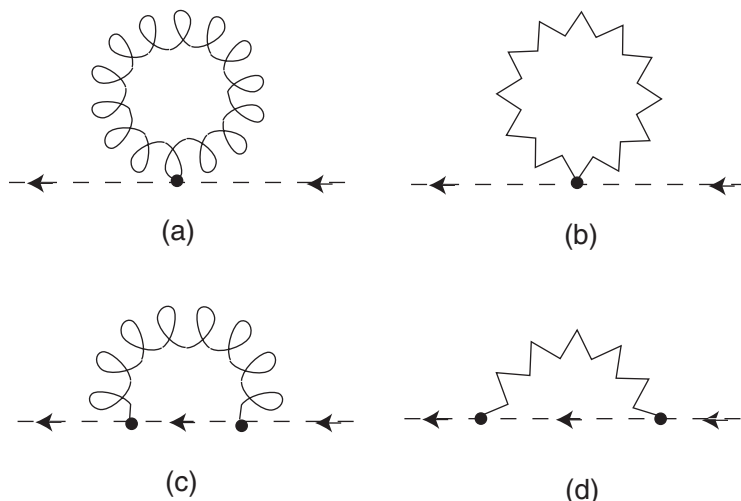
where  $L$  is the number of loops,  $E$  is the number of external scalar lines,  $E'$  is the number of external vector lines, and  $E_g$  is the number of external ghosts. [For arbitrary  $N$ , one finds  $d = 2(N + 1) - 2(N - 1)L - E - E' - N E_g$ .] For the gauge boson and complex scalar self-energies,  $d = 6 - 4L$ ; the divergences are at most quadratic and occur at no higher than one loop.

In the case of the gauge boson self-energies, the cancellation of the potential quadratic divergence is a consequence of gauge invariance, as in the  $N = 2$  theory [3]. Amplitudes in the HD theory satisfy a Ward identity, which implies that the 1-particle irreducible two-point function for  $\hat{A}$  must be of the form  $(q^2 g_{\mu\nu} - q_\mu q_\nu)$  times a dimensionless function of the regulator scale and the external momentum  $q^2$ . A straightforward power counting of HD Lagrangian mass parameters shows that they only multiply the divergent parts of the possible one-loop diagrams in dimensionless ratios, so that the divergence is at most logarithmic. An equivalent calculation in the LW form of the Lagrangian is possible but prohibitive in theories with  $N > 2$  due to the proliferation of gauge boson self-interactions [see, for example, eq. (3.13)]. If a chiral fermion is added to the  $N = 3$  theory, one finds that the fermion-vector coupling scales as  $p^4$ , the fermion/two-vector coupling scales as  $p^3$ , and the fermion propagator as  $p^{-5}$ . It follows immediately that the one-loop fermion contributions to the gauge boson self-energy have  $d = 2$ ; the quadratic divergence cancels for the same reason as in the purely bosonic loop diagrams.

In the case of the complex scalar, on the other hand, it is straightforward to show the cancellation of one-loop divergences in the LW form of the theory. We present the explicit calculation below as an illustration of the formalism.

### 6.1 The ordinary scalar

We first consider the mass renormalization of the ordinary complex scalar field  $H_1$ . The  $\eta_i$  shown in the formulae below are functions eqs. (2.29)–(2.31) of the gauge boson masses  $m_1 = 0$ ,  $m_2$  and  $m_3$ . We make the same assumptions as ref. [3], that the scalar potential is vanishing so that the ordinary scalar is massless, and work in Feynman gauge.



**Figure 1:** Diagrams that contribute to the mass renormalization of the complex scalars. The dashed lines refer to the field  $H^{(i)}$ , for  $i = 1, 2$  or  $3$ . The curly lines represent the ordinary gauge field  $A^{(1)}$ ; the zigzag lines represent its LW partners  $A^{(2)}$  or  $A^{(3)}$ .

Equations (32a)-(32d) in ref. [3] generalize as follows:

$$-i\Sigma_a(0) = g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \frac{n}{k^2}, \tag{6.3}$$

$$i\Sigma_b(0) = -g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \left[ \left( \frac{n-1}{k^2 - m_2^2} - \frac{1}{m_2^2} \right) \left( -\frac{\eta_2}{\eta_1} \right) - \left( \frac{n-1}{k^2 - m_3^2} - \frac{1}{m_3^2} \right) \left( \frac{\eta_3}{\eta_1} \right) \right], \tag{6.4}$$

$$-i\Sigma_c(0) = -g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2}, \tag{6.5}$$

$$-i\Sigma_d(0) = -g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \left[ \frac{1}{m_2^2} \left( -\frac{\eta_2}{\eta_1} \right) - \frac{1}{m_3^2} \left( \frac{\eta_3}{\eta_1} \right) \right]. \tag{6.6}$$

These results correspond to the diagrams shown in figure 1. The cancellation of quartic divergences [between eqs. (6.4) and (6.6)] is obvious by inspection. The quadratic divergence originates from

$$\frac{n}{k^2} + \frac{n-1}{k^2} \left( \frac{\eta_2 + \eta_3}{\eta_1} \right) - \frac{1}{k^2}, \tag{6.7}$$

where the terms are the  $k^2 \gg m_i^2$  limits of the integrands of eqs. (6.3), (6.4) and (6.5), respectively. This quantity vanishes because  $\eta_1 + \eta_2 + \eta_3 = 0$ . Hence, the ordinary scalar mass remains logarithmically divergent, as in the  $N = 2$  theory.

## 6.2 The Negative-Norm LW scalar

The normal scalar discussed in the last subsection has two LW partners in the  $N = 3$  theory. We first consider the shift in the pole mass of the lighter, negative-norm state,



whose mass we denote by  $m_{H_2}$ . Equations (33a)-(33d) in ref. [3] generalize as follows:

$$-i\Sigma_a(m_{H_2}^2) = -g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \frac{n}{k^2}, \quad (6.8)$$

$$-i\Sigma_b(m_{H_2}^2) = g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \left[ \left( \frac{n-1}{k^2 - m_2^2} - \frac{1}{m_2^2} \right) \left( -\frac{\eta_2}{\eta_1} \right) - \left( \frac{n-1}{k^2 - m_3^2} - \frac{1}{m_3^2} \right) \left( \frac{\eta_3}{\eta_1} \right) \right], \quad (6.9)$$

$$-i\Sigma_c(m_{H_2}^2) = g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \left[ \frac{1}{k^2 - 2p \cdot k} + \frac{4m_{H_2}^2 - 4p \cdot k}{k^2(k^2 - 2p \cdot k)} \right], \quad (6.10)$$

$$-i\Sigma_d(m_{H_2}^2) = g^2 C_2(N_c) \int \frac{d^n k}{(2\pi)^n} \left[ \left( \frac{1}{m_2^2} - \frac{4m_{H_2}^2 - 2p \cdot k}{(k^2 - m_2^2)(k^2 - 2p \cdot k)} \right) \left( -\frac{\eta_2}{\eta_1} \right) - \left( \frac{1}{m_3^2} - \frac{4m_{H_2}^2 - 2p \cdot k}{(k^2 - m_3^2)(k^2 - 2p \cdot k)} \right) \left( \frac{\eta_3}{\eta_1} \right) \right]. \quad (6.11)$$

Terms manifestly odd in  $k$  have been dropped. Quartically divergent terms clearly cancel between eqs. (6.9) and (6.11). Quadratic divergences are found in eqs. (6.8), (6.9) and (6.10), but again in a combination proportional to  $\eta_1 + \eta_2 + \eta_3 = 0$ . Thus, quadratic divergences cancel between diagrams and only a logarithmic divergence remains.

### 6.3 The Positive-Norm LW scalar

The on-shell self-energies of the heavier, positive-norm LW scalar (with mass  $m_{H_3}$ ) may be obtained from eqs. (6.8)–(6.11) by replacing  $m_{H_2} \rightarrow m_{H_3}$ , and by flipping the overall sign of these results. The sign flip originates from the change in sign of the  $H_3$  quadratic terms relative to those of  $H_2$ . In the  $a$  and  $b$  diagrams, the sign flip originates from the opposite sign of the two-scalar/two-gauge vertex; in the  $c$  and  $d$  diagrams, it originates from sign changes at each vertex and in the scalar propagator. These modification do not alter the cancellation of divergences between diagrams, so that the positive-norm LW scalar mass also receives only logarithmic corrections.

### 6.4 Yukawa couplings

If chiral fermions are present in the theory, then one may also consider the effect of Yukawa couplings like

$$\mathcal{L} = \lambda \left( \hat{\phi}_L \hat{H} \hat{\psi}_R + \text{h.c.} \right), \quad (6.12)$$

where  $\hat{\phi}_L$  transforms in the fundamental representation, while  $\psi_R$  is a singlet. Letting  $\eta_i$  refer to the LW mass spectrum of  $\phi_L^{(i)}$  and  $\eta'_i$  to that of  $\psi_R^{(i)}$ , it is easy to see that the quadratically divergent part of the one-fermion loop contribution to the complex scalar self-energy is proportional to

$$\left( 1 + \frac{\eta_2}{\eta_1} + \frac{\eta_3}{\eta_1} \right) \left( 1 + \frac{\eta'_2}{\eta'_1} + \frac{\eta'_3}{\eta'_1} \right) \quad (6.13)$$

which vanishes since  $\eta_1 + \eta_2 + \eta_3 = 0$  (and similarly for the  $\eta'_i$ ), again confirming that the quadratic divergences are cancelled at one loop.

## 7. Conclusions

The Lee-Wick Standard Model provides a new theory that is interesting from both the formal field-theoretical and phenomenological points of view. Its means of solving the hierarchy problem, by cancelling the leading divergences of loop diagrams between each particle and a partner of the same statistics and quantum numbers but carrying wrong-sign kinetic and mass terms, is innovative and worthy of detailed study.

To this end, we have developed the generalization of the theory to allow each particle two LW partners. Since the original Lee-Wick Standard Model [3] involves higher-derivative quadratic terms of  $O(p^4)$  in momentum space (for the bosonic fields), our theory necessarily includes terms of  $O(p^6)$ . Referring to the number of poles in the two-point function, we name these the  $N = 2$  and  $N = 3$  Lee-Wick theories, respectively. We note that there is no impediment, in principle, that prevents the generalization of our approach to theories with  $N > 3$ .

The recasting of HD theories in terms of fields satisfying low-order equations of motion (the Ostrogradsky method for reducing high-order differential equations to a recursive system of low-order ones, as applied to quantum field theory) was developed decades ago by Pais and Uhlenbeck [12]. The results presented here are new in a number of significant respects. First, we supply the prescription for rewriting a viable  $N = 3$  HD theory in terms of an equivalent AF theory containing no terms of dimension higher than four; the  $N = 2$  case was developed of course by Grinstein *et al.* in ref. [3]. Such auxiliary fields provide constraints that are exact at the quantum level, and once imposed, exactly reproduce the HD Lagrangian. On the other hand, the auxiliary fields may be rewritten in terms of a set of fields whose quadratic terms are canonical, up to overall signs, and whose couplings are intricately intertwined. For  $N = 3$ , these fields consist of the original particle, one negative-norm and one positive-norm LW partner; the three fields together conspire to cancel the quadratic divergences in the theory. Notably, our  $N = 3$  analysis includes non-Abelian chiral gauge theories, with or without spontaneous symmetry breaking, topics that were not addressed in the ancient literature on nonlocal Lagrangians.

We have successfully developed this construction, with minor variations, in theories with real scalars, fermions, gauge bosons, and complex scalars, and allowing for spontaneous symmetry breaking. One concludes that the entire Standard Model may be easily embedded in an  $N = 3$  LW theory, a possibility that offers an abundant new wellspring for future studies of the formal properties and phenomenology of the model.

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